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Magnetoacoustic Attenuation by Bragg-Reflected Electrons*

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We discuss the difficulties associated with the jellium model and the validity of the deformation-potential method of calculating magnetoacoustic attenuation in metals when the electrons are Bragg reflected. We then calculate the attenuation of a transverse acoustic wave propagating along the magnetic field in a metal whose Fermi surface consists of a free-electron sphere truncated by the six Bragg planes of a simple-cubic Brillouin zone. The attenuation-vs-magnetic-field curves show considerable structure which depends strongly on the ratio $K/2k_F$ where K is the separation between Bragg planes.

I. INTRODUCTION

The jellium theory of magnetoacoustic attenuation[1,2] was extended to nonspherical Fermi surfaces by Eckstein.[3] She was attempting to reproduce peaks found by Boyd and Gavenda[4] in the magnetic field dependence of the attenuation of transverse acoustic waves propagating parallel to the magnetic field along the [100] direction in copper. They attributed these peaks to singularities in the $v_H = \partial E/\partial p_H$ density of states on the Fermi surface. Eckstein chose a dumbbell-shaped model Fermi surface with such singularities and found a peak in the conductivity which leads to a *dip* in the attenuation. Thinking this might be a consequence of the jellium theory, which strictly speaking could be valid only for spherical Fermi surfaces, we recalculated[5] the attenuation using a free-electron deformation-potential (FED) approximation to the exact deformation potential.[6,7] We found, however, that the FED approximation gave exactly Eckstein's jellium result for her Fermi surface and for more complicated dumbbell-shaped Fermi surfaces gave results more compli-

cated than the jellium theory but which were numerically still almost identical to the jellium results. The use of the exact deformation potential (which can be calculated only for a real material, and not from a model Fermi surface) would destroy the numerical agreement with the jellium calculation but not the general agreement in the shape of the attenuation-vs-magnetic-field curves.

In this paper we examine the effect of Bragg reflections on the attenuation of transverse sound waves propagating along the magnetic field direction. We choose a Fermi surface consisting of a free-electron sphere truncated by the six Bragg planes of a simple-cubic Brillouin zone. [See Fig. 1(a).] For $K/2k_F > 1$ (where K is the separation between Bragg planes) we have simply the free-electron sphere. For $1 \geq K/2k_F \geq 1/\sqrt{2}$, the electrons at the top and bottom of the sphere are missing, the electrons in the middle of the sphere are Bragg reflected four times in a single orbit, and the remaining electrons keep their free-electron orbits. In Fig. 1(b) we show a Bragg-reflected orbit. We shall refer to the Bragg-reflected electrons as holes because the Bragg-reflected orbit shown in Fig. 1(b) is identical to the hole orbit shown in Fig. 1(c). For $1/\sqrt{2} \geq K/2k_F \geq 1/\sqrt{3}$, the middle as well as the top and bottom of the sphere lie entirely outside the first Brillouin zone and the only remaining orbits are hole orbits.

Sievert[8] has studied Bragg-reflected orbits (open orbits in his case) using the jellium theory. In a Note Added in Proof he states that Blount has pointed out the omission of the impulsive term responsible for the Bragg reflection in the force felt by the electrons. It is not completely clear to us what form this term should take since a Bragg reflection does not occur off a single plane, but rather is the result of the interference of several planes. Fortunately in the deformation-potential theory this term cannot ever occur. (Remember that the deformation potential is derived by transforming to coordinates at rest in the ions.[9] Therefore the electrons Bragg-reflect off stationary planes of ions.) Another more subtle difficulty also arises in the jellium theory. There is a collision-drag contribution[1,2] to the attenuation of the form $N_{eff} m(\vec{u}_t - \langle\vec{v}\rangle)\cdot\vec{u}_t^*$, where $\vec{u}_t$ is the ionic velocity, $\langle\vec{v}\rangle$ is the electron velocity averaged over the free part of the Fermi surface (i.e., not over that part consisting of Bragg planes) and N_{eff} is an integral over the free part of the Fermi surface which reduces to N, the number of conduction electrons, when there are no Bragg planes. $-N_{eff} e\langle\vec{v}\rangle$ is the electronic current which is normally very nearly equal and opposite to the ionic current, $\vec{J} = Ne\vec{u}_t$. Thus when there are no Bragg planes and $N_{eff} = N$, the collision-drag term is negligible but in the present case it is not. Sievert[8] included the collision-drag term but failed to distinguish between N and N_{eff}. One might also argue that $\vec{J}$ should be replaced by an effective ionic current because those electrons which are blocked by the Bragg planes from contributing to the conductivity are still effective in screening the ionic current.[10] Fortunately in the deformation-potential method there is no ionic current and no collision-drag term, so these questions do not arise. It should be emphasized that the deformation-potential theory does not neglect these terms; they just do not enter explicitly. The deformation-potential results[5] for the free-electron and Eckstein[3] Fermi surfaces contain terms which in the jellium derivation arise from collision drag[11] even though collision drag never entered the deformation-potential calculation.

In Sec. II of this paper we derive the attenuation for the geometry of Fig. 1 and in Sec. III we present the numerical calculations and give physical explanations for the peaks and dips appearing in the calculated curves.

II. ATTENUATION WITH BRAGG REFLECTIONS

We here calculate the attenuation of an acoustic wave using the FED method of Ref. 5 (hereafter called I). It should be pointed out that our Fermi surface of Fig. 1 is somewhat unphysical in that the energy gap is assumed to be sufficiently large that there are no occupied states in the second Brillouin zone but the crystal potential is sufficiently weak that the Fermi surface in the first zone is completely free-electron-like except for the Bragg planes. However, once given this free-electron-like Fermi surface, the FED approximation becomes exact. From Eq. (I5) we have for the deviation of the electronic distribution function from equilibrium[12]

$$f_1 = \int_{-\infty}^{t} \frac{\partial f_0}{\partial E}\,(e\vec{\mathcal{E}}'\cdot\vec{v}' + i\omega\vec{\vec{\epsilon}}' : \tilde{D}')e^{-(t-t')/\tau}dt' , \tag{1}$$

where $\vec{\mathcal{E}}$ is the electric field induced by the elec-

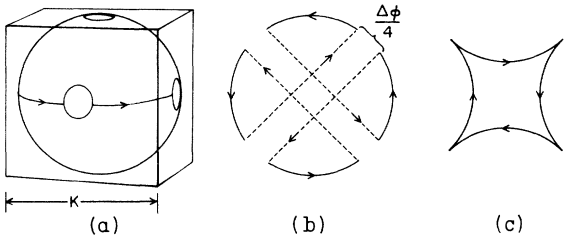

FIG. 1. (a) Free-electron Fermi surface truncated by the six Bragg planes of a simple-cubic Brillouin zone. (b) Orbit of a Bragg-reflected electron. (c) Identical orbit shown as hole orbit.

tron's response to $\tilde{D}$, the deformation-potential tensor, $\overleftrightarrow{\epsilon}$ is the acoustic strain tensor, τ the electron relaxation time, and primed quantities are evaluated at t'. In the FED [Eq. (I9)] we have

$$\overleftrightarrow{\epsilon} : \tilde{D} = -\hbar \vec{k} \cdot \overleftrightarrow{\epsilon} \cdot \vec{v} , \tag{2}$$

where $\vec{k}$ and $\vec{v}$ are the electron's wave vector and velocity. Using $\overleftrightarrow{\epsilon} = \partial \vec{u}/\partial \vec{r} = i\vec{u}\vec{q}$ where the ionic displacement $\vec{u} = \hat{i} u_\perp e^{i(\vec{q}\cdot\vec{r}-\omega t)}$, we have from (I12) and (I15)

$$\vec{\mathcal{E}}' \cdot \vec{v}' = v_\perp [\mathcal{E}_+ \cos(\vec{q}\cdot\vec{r}' - \Omega_+ t' - \varphi') + \mathcal{E}_- \cos(\vec{q}\cdot\vec{r}' - \Omega_- t' + \varphi')] , \tag{3}$$

$$\overleftrightarrow{\epsilon}' : \tilde{D}' = -i\tfrac{1}{2}\hbar v_z q k_\perp v_\perp [\cos(\vec{q}'\cdot\vec{r}' - \Omega_+ t' - \varphi') + \cos(\vec{q}\cdot\vec{r}' - \Omega_- t' + \varphi')] , \tag{4}$$

where $\Omega_\pm = \omega \pm \omega_c$, $\varphi' = \varphi - \omega_c t - n(\pi + \Delta\varphi/4)$, and $\omega_c = eH/mc$. φ is the azimuthal angle of $\vec{k}(t)$ and n the number of Bragg reflections a hole has suffered between t and t'. Note from Fig. 1(b) that the phase angle is increased by $\pi + \frac{1}{4}\Delta\varphi$ on each Bragg reflection.[13] In Appendix A we perform the integration over t' and obtain

$$f_1 = \frac{\partial f_0}{\partial E} \frac{\tau v_\perp}{d_\pm} (e\mathcal{E}_\pm + \tfrac{1}{2}\omega m q v_z u_\perp)(1 - Q_\pm e^{-d_\pm (t-t_1)/\tau}) \times e^{i(\vec{q}\cdot\vec{r} - \omega t \mp \varphi)} , \tag{5}$$

where t_1 is the time at which the hole executed its last Bragg reflection,

$$d_\pm = 1 + i(q v_z - \Omega_\pm)\tau , \tag{6}$$

$$Q_\pm = \frac{e^{\pm i\Delta\varphi/4} + 1}{e^{\pm i\Delta\varphi/4 - d_\pm \Phi/\omega_c\tau} + 1} \quad \text{(holes)}, \tag{7}$$

$$Q_\pm = 0 \quad \text{(electrons)}, \qquad \Phi = (\tfrac{1}{2}\pi - \Delta\varphi/4) , \tag{8}$$

and a sum over (±), i.e., over both senses of circular polarization, is implied in (5). Thus the distribution function for holes is the sum of two terms; the first is identical to the distribution function for electrons and the second is due to the effect of Bragg reflections.

The next task is to evaluate $\mathcal{E}_\pm$. The electronic (and hole) current is obtained from

$$\vec{j} = -2(2\pi)^{-3} e \int \vec{v} f_1 d^3k \tag{9}$$

in Appendix B:

$$j_\pm = \frac{2e\tau m}{(2\pi\hbar)^2} \left[e\mathcal{E}_\pm \int \frac{v_\perp^2}{d_\pm} F_\pm(k_z)\, dk_z - \frac{im\omega u_\perp}{2\tau} \times \left(\int v_\perp^2 F_\pm(k_z)\, dk_z - b_\pm \int \frac{v_\perp^2}{d_\pm} F_\pm(k_z)\, dk_z \right) \right] = 2\sigma_\pm \left(\mathcal{E}_\pm G_\pm - \frac{i\omega m u_\perp}{2e\tau} (1 - b_\pm G_\pm) \right) , \tag{10}$$

where

$$F_\pm(k_z) = (1 - \Delta\varphi/2\pi) + (2\omega_c\tau/\pi d_\pm) Q_\pm (e^{-d_\pm \Phi/\omega_c\tau} - 1) , \tag{11}$$

$$b_\pm = 1 - i\Omega_\pm \tau , \tag{12}$$

$$\sigma_\pm = e^2 \tau N_\pm / m , \tag{13}$$

$$N_\pm = (m/2\pi\hbar)^2 \int v_\perp^2(k_z) F_\pm(k_z)\, dk_z , \tag{14}$$

$$G_\pm = \int (v_\perp^2/d_\pm) F_\pm(k_z)\, dk_z \Big/ \int v_\perp^2 F_\pm(k_z)\, dk_z . \tag{15}$$

Note that $F_\pm(k_z) = 1$ for electrons. Note also that if there is no magnetic field $\omega_c = 0$ and

$$N_\pm = m^2(2\pi\hbar)^{-2} \int v_\perp^2 (1 - \Delta\varphi/2\pi)\, dk_z$$

is just the effective number of electrons and holes and $\sigma_\pm$ is the ordinary dc conductivity. In this case the carriers do not precess (and hence do not Bragg reflect), and there is no distinction between the electrons and holes insofar as their contribution to the conductivity is concerned. From Maxwell's equations we have[2]

$$\mathcal{E}_\pm = \frac{4\pi i}{\omega} \left(\frac{v_s}{c} \right)^2 j_\pm , \tag{16}$$

where v_s is the velocity of the acoustic wave. Eliminating $j_\pm$ between (10) and (16) we have

$$e\mathcal{E}_\pm = \frac{i\omega m u_\perp}{2\tau} \frac{1 - b_\pm G_\pm}{G_\pm + i\beta_\pm} , \tag{17}$$

where

$$\beta_\pm = \frac{\omega}{8\pi\sigma_\pm} \left(\frac{c}{v_s} \right)^2 . \tag{18}$$

The distribution function then becomes

$$f_1 = \frac{\partial f_0}{\partial E} \frac{im\omega v_\perp u_\perp}{2d_\pm} \left(\frac{1 + ib_\pm\beta_\pm}{G_\pm + i\beta_\pm} - d_\pm \right) \times [1 - Q_\pm e^{-d_\pm (t-t_1)/\tau}]\, e^{i(\vec{q}\cdot\vec{r} - \omega t \mp \varphi)} . \tag{19}$$

The total electronic excitation energy due to the acoustic wave is given by (I31):

$$E = (2\pi)^{-3} \int f_{1\pm} \tfrac{1}{2} J_\pm^*\, d^3k , \tag{20}$$

where $J_\pm$ is just $f_{1\pm}$ with the factor $\partial f_0/\partial E$ missing. Using (I20), $d^3k = m\hbar^{-2} d\varphi\, dk_z\, dE$, and $\partial f_0/\partial E = -\delta(E - E_F)$, one obtains

$$E=\frac{m}{2(2\pi)^3\hbar^2}\left(\frac{\omega m u_\perp}{2}\right)^2\int v_\perp^2\left(\frac{|1\pm ib_\pm\beta_\pm|^2}{|G_\pm+i\beta_\pm|^2|d_\pm|^2}-\frac{1+ib_\pm\beta_\pm}{d_\pm(G_\pm+i\beta_\pm)}-\frac{1-ib_\pm^*\beta_\pm^*}{d_\pm^*(G_\pm^*-i\beta_\pm^*)}+1\right)$$
$$\times\left[1-e^{-d_\pm(t-t_1)/\tau}Q_\pm-e^{-d_\pm^*(t-t_1)/\tau}Q_\pm^*+\left|e^{-d_\pm(t-t_1)/\tau}Q_\pm\right|^2\right]d\varphi\,dk_z\,. \quad (21)$$

Noting that $1/|d_\pm|^2=\mathrm{Re}\,(1/d_\pm)$ this may be written

$$E=\frac{m}{2(2\pi)^3\hbar^2}\left(\frac{\omega m u_\perp}{2}\right)^2\mathrm{Re}\left\{\frac{|1+ib_\pm\beta_\pm|^2}{|G_\pm+i\beta_\pm|^2}\left[\int\frac{v_\perp^2}{d_\pm}\left(1-e^{-d_\pm(t-t_1)/\tau}Q_\pm\right)d\varphi\,dk_z\right.\right.$$
$$\left.+\int\frac{v_\perp^2}{d_\pm}\left(\left|e^{-d_\pm(t-t_1)/\tau}Q_\pm\right|^2-e^{-d_\pm^*(t-t_1)/\tau}Q_\pm^*\right)d\varphi\,dk_z\right]-2\frac{1+ib_\pm\beta}{G_\pm+i\beta}\left[\int\frac{v_\perp^2}{d_\pm}\left(1-e^{-d_\pm(t-t_1)/\tau}Q_\pm\right)d\varphi\,dk_z\right.$$
$$\left.\left.+\int\frac{v_\perp^2}{d_\pm}\left(\left|e^{-d_\pm(t-t_1)/\tau}Q_\pm\right|^2-e^{-d_\pm^*(t-t_1)/\tau}Q_\pm^*\right)d\varphi\,dk_z\right]\right.$$
$$\left.+\int v_\perp^2\left(1-e^{-d_\pm(t-t_1)/\tau}Q_\pm\right)d\varphi\,dk_z+\int v_\perp^2\left(\left|-e^{-d_\pm(t-t_1)/\tau}Q_\pm\right|^2-e^{-d_\pm^*(t-t_1)/\tau}Q_\pm^*\right)d\varphi\,dk_z\right\}\,. \quad (22)$$

We note that the entire φ dependence of the integrand is in $(t-t_1)$, which may be replaced by φ/ω_c and integrated over $0\le\varphi\le\Phi$ in each of the four sections of the hole orbit; for electron orbits there is no φ dependence and the φ integration yields a factor of 2π. Performing the integration and using (7), the second term in the first set of square brackets can be shown to be pure imaginary and thus not contribute to E. Finally one has

$$E=\frac{m\omega}{2(2\pi)}\left(\frac{u_\perp}{2}\right)^2$$
$$\times\mathrm{Re}\left\{N_\pm\left[\frac{|1+ib_\pm\beta_\pm|^2}{|G+i\beta_\pm|^2}G_\pm-2\frac{1+ib_\pm\beta_\pm}{G_\pm+i\beta_\pm}G_\pm+1\right]\right.$$
$$\left.+N_\pm'\left[-2\frac{1+ib_\pm\beta_\pm}{G_\pm+i\beta_\pm}G_\pm'+1\right]\right\}\,, \quad (23)$$

where

$$N_\pm'=(m/2\pi\hbar)^2\int F_\pm'(k_z)\,v_\perp^2\,dk_z\,, \quad (24)$$

$$G_\pm'=\left(\int(v_\perp^2/d_\pm)F_\pm'(k_z)\,dk_z\right)/\int v_\perp^2F_\pm(k_z)\,dk_z\,, \quad (25)$$

and

$$F_\pm'(k_z)=2\omega_c\tau\left[\left(2Q_\pm^*/d_\pm^*\right)\left(e^{-d_\pm^*\Phi/\omega_c\tau}-1\right)-\left|Q_\pm\right|^2\right.$$
$$\left.\times\left(e^{-2\Phi/\omega_c\tau}-1\right)\right]\,. \quad (26)$$

This excitation energy is converted to heat with a time constant $\frac{1}{2}\tau$. Dividing by the incident-phonon energy flux $M\omega^2(\frac{1}{2}u_\perp)^2v_s$ where M is the density of the metal, we obtain for the attenuation coefficient

$$\alpha_\pm=\frac{m}{Mv_s\tau}$$
$$\times\mathrm{Re}\left[N_\pm\left(\frac{|1+ib_\pm\beta_\pm|^2}{|G_\pm+i\beta_\pm|^2}G_\pm-2\frac{1+ib_\pm\beta_\pm}{G_\pm+i\beta_\pm}G_\pm+1\right)\right.$$
$$\left.+N_\pm'\left(1-2\frac{1+ib_\pm\beta_\pm}{G_\pm+i\beta_\pm}G_\pm'\right)\right]\,. \quad (27)$$

It was assumed in I that β is negligible; this corresponds to nearly perfect screening of the ionic current (in the lab frame) by the electrons or holes and is valid except for very high frequency, very large magnetic field, or when $K/2k_F\to 1/\sqrt{3}$ so that there are only a few carriers. Thus we have

$$\alpha_\pm\approx\frac{m}{Mv_s\tau}\mathrm{Re}\left[N_\pm\left(\frac{1}{G_\pm}-1\right)+N_\pm'\left(1-2\frac{G_\pm'}{G_\pm}\right)\right]\,. \quad (28)$$

The first term in (28) looks like the result for a spherical Fermi surface; however, the Bragg reflections have completely modified $N_\pm$ and $G_\pm$. N_- is no longer the density of electrons although at zero field (when no Bragg reflections occur) it is the effective density of electrons. At finite fields it loses even this property and becomes a complex quantity. Similarly at zero field $G_\pm$ is like the free electron $G_\pm$ but is calculated only from the free part of the Fermi surface. The second term which vanishes at zero field is due completely to Bragg reflections.

III. RESULTS AND DISCUSSION

We have numerically evaluated $\alpha_\pm$ from Eq. (27) neglecting ω in comparison with ω_c (which is valid for all but the smallest magnetic fields and makes $\alpha_+=\alpha_-$). Note that the entire k_z dependence of the integrand arises from v_z, $v_\perp$, and $\Delta\varphi$.[13] The α-vs-$\gamma=\omega_c/qv_F$ curves are displayed in Figs.

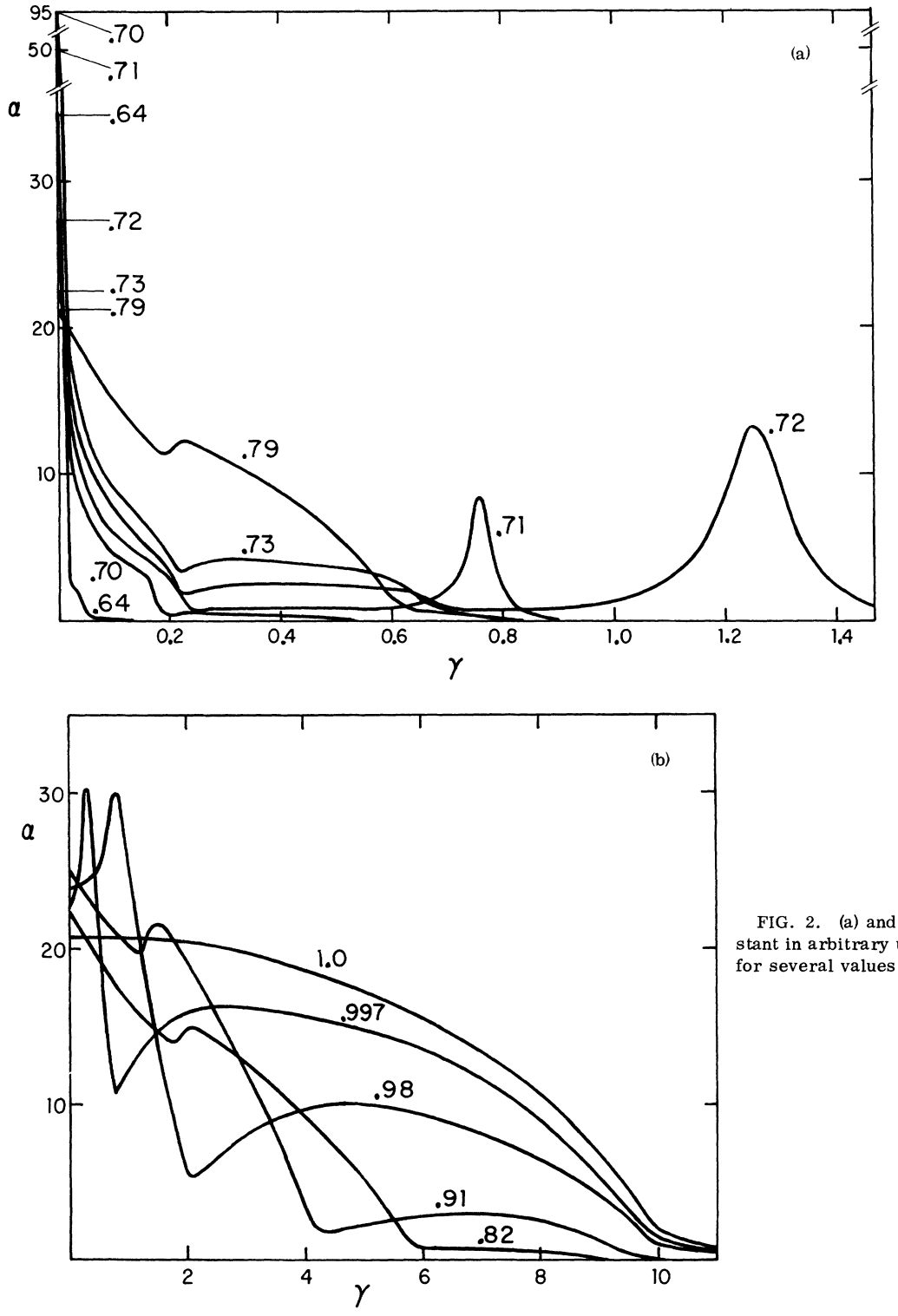

FIG. 2. (a) and (b) Attenuation constant in arbitrary units vs $\gamma = \omega_c/qV_F$ for several values of $K/2k_F$.

2(a) and 2(b) for several values of $K/2k_F$ with $qv_F\tau = 50$. Interesting structure occurs when there are a large number of electrons and a few holes ($K/2k_F$ slightly less than unity) or when there are a large number of holes and a few electrons ($K/2k_F$ slightly larger than $1/\sqrt{2}$). In Fig. 3 we repeat the attenuation curve for $K/2k_F = 0.98$ and compare it with the attenuation when the holes

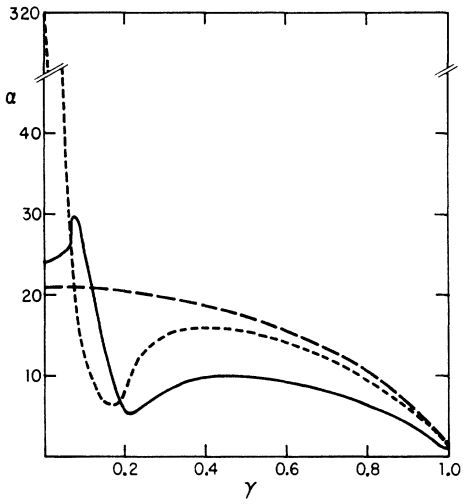

FIG. 3. Comparison of the attenuation when $K/2k_F = 0.98$ (solid curve) with that for the full Fermi sphere (long dashes) and with the attenuation when $K/2k_F = 0.98$ but with the hole orbits absent (short dashes).

are completely absent, i.e., for a Fermi surface consisting of two spherical electron caps with $0.2 \leq |k_z| \leq 1$ and also with the attenuation for the full spherical Fermi surface, i.e., for $K \geq 2k_F$. We note a huge peak at $\omega_c = 0$ and a large dip centered at $\omega_c/qv_F = 0.17$ for the electron-cap Fermi surface. This can easily be understood as follows. The electron attenuation is proportional to $N^2/\bar{G}$ where $\bar{G} = G/N$ [see Eqs. (14) and (15)]. Now $\bar{G}$ is contributed to almost entirely by electrons which obey the resonance condition $qv_z = \omega_c$ which makes the imaginary part of d in Eq. (6) vanish. Thus for small ω_c the resonant electrons are absent and $\bar{G}$ is extremely small. For large ω_c the resonant electrons are present and $\bar{G}$ is almost identical to its value for the full electron sphere. On the other hand, N is independent of ω_c and is reduced from its value for the full electron sphere. Thus the behavior of $N^2/\bar{G}$ explains this attenuation curve.[14]

Whereas in Fig. 3 we showed the attenuation when the holes were completely absent, in Fig. 4 we show the attenuation due to only the holes and also the attenuation which would be obtained if those same holes were electrons, i.e., neglecting the effect of Bragg reflections. Because of the missing part of their orbits $\Delta\varphi$, the holes execute an orbit in less time than electrons and according to Fig. 1(c) with an opposite sense of rotation so that one would expect the resonance condition to be

$$qv_z = \omega_e = \frac{-\omega_c}{1 - \Delta\varphi/2\pi} \ . \tag{29}$$

If one follows the Bragg-reflecting electron around Fig. 1(b) as well as the phase of the Doppler-shifted acoustic wave with which it is resonant, one sees that they are in phase over two of the legs of the orbit and out of phase over the other two, so one would expect the absorption due to holes to be less than that due to electrons, as Fig. 4 indeed shows. Furthermore, one might expect harmonics of this resonance to occur. This resonance and its harmonics can be traced back to peaks in $Q_\pm$ which occur when the imaginary part of the argument of the exponential in Eq. (7) is equal to an odd integer number of π. This occurs when

$$qv_z = (-1)^n(2n-1)\omega_e \ , \tag{30}$$

where n is a positive integer. Since for the extremal hole orbit $v_z = 0.2v_F$ and $\omega_e = -\omega_c$, the attenuation vanishes when $\omega_c/qv_F > 0.2$, i.e., when there are no holes which can satisfy the resonance condition. When $\omega_c/qv_F < 0.2/(2n-1)$ there are holes with n different values of v_z which satisfy n different odd harmonics. Thus in Fig. 4 we show points numbered between 4 and 0 such that if ω_c/qv_F lies between two points, the number on the left is the number of harmonics satisfied. The total absorption for $K/2k_F = 0.98$ can then be understood as a combination of the absorption of the holes alone and of the electrons alone.

When $K/2k_F$ is just slightly larger than $1/\sqrt{2}$, there are two small slabs of electron Fermi surface with $v_z \approx \pm v_F/\sqrt{2}$ and a large hole section of Fermi surface with v_z lying between the electron values of v_z. If ω_c/qv_F is greater than $1/\sqrt{2}$ then both the electrons and holes are all below resonance

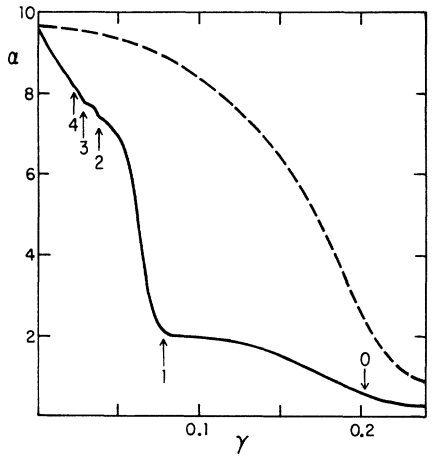

FIG. 4. Attenuation due to the holes alone when $K/2k_F = 0.98$ (solid curve) and due to those same holes but neglecting their Bragg reflections (dashed curve). The numbered arrows indicate that point below which an odd harmonic of the resonance condition is satisfied by the holes.

and G_1 is small [where $\mathrm{Re}(1/G) = G_1/(G_1^2+G_2^2)$]. The sign of G_2 is opposite for the electrons and holes. Although there are many fewer electrons than holes, the electrons are closer to resonance (because of their larger v_z) and the electron and hole contributions to G_2 are of the same order of magnitude. For some particular value of ω_c/qv_F, they will have exactly the same magnitude and G_2 will vanish, causing a peak in $\mathrm{Re}(1/G)$. The fewer the number of electrons, the closer they must be to resonance for G_2 vanish, i.e., the closer ω_c/qv_F must be to $1/\sqrt{2}$. This explains the peaks centered at $\omega_c/qv_F = 0.76$ and 1.25 for $K/2k_F = 0.71$ and 0.72. The very sharp peaks both curves have at $\omega_c = 0$ are due to the very small number of $v_z = 0$ resonant holes. When $K/2k_F \to 1/\sqrt{2}$, $\Delta\varphi(k_z = 0) \to 2\pi$ and the $k_z = 0$ holes disappear altogether. This peak, caused by a reduction in the number of resonant carriers, corresponds closely to the dip in attenuation found by Eckstein[3] when the number of resonant electrons on her Fermi surface was singular.

Although our simple-model Fermi surface does not reproduce the magnetoacoustic attenuation of the complicated copper Fermi surface, we have for the first time shown how the introduction of Bragg planes to create hole orbits can lead to peaks in the attenuation and hence have made theory and experiment at least compatible.

APPENDIX A

We here perform the integration over t' in Eq. (1) for holes (the integration being trivial for electrons). Replacing the cosines in (3) and (4) by exponentials, we obtain integrals of the form

$$\int_{-\infty}^{t} \chi(t',\varphi')\,dt' = \int_{-\infty}^{t_1} \chi(t',\varphi')\,dt' + \int_{t_1}^{t} \chi(t',\varphi')\,dt' , \tag{A1}$$

where

$$\chi(t',\varphi') = e^{i[(qv_z-\Omega_\pm)t' \mp \varphi'] - (t-t')/\tau} , \tag{A2}$$

and t_1 is the time at which the last Bragg reflection occurred for the hole whose phase is φ at time t. Now

$$\begin{aligned}\int_{-\infty}^{t_1} \chi(t',\varphi')\,dt' &= \int_{-\infty}^{t_2} \chi(t',\varphi')\,dt' + \int_{t_2}^{t_1} \chi(t',\varphi')\,dt' \\ &= \int_{-\infty}^{t_1} \chi[t''-T, \varphi'' - (\pi + \Delta\varphi/4)]\,dt'' \\ &\quad + \int_{t_2}^{t_1} \chi(t',\varphi')\,dt' ,\end{aligned} \tag{A3}$$

where

$$T = t_1 - t_2 = (\tfrac{1}{2}\pi - \Delta\varphi/4)/\omega_c \tag{A4}$$

is the time between successive Bragg reflections and $(\pi + \Delta\varphi/4)$ is the phase shift due to a Bragg reflection [see Fig. 1(b)]. Thus we have

$$\int_{-\infty}^{t_1} \chi(t',\varphi')\,dt' = (1 + e^{-d_\pm T/\tau \pm i\Delta\varphi/4})^{-1} \int_{t_2}^{t_1} \chi(t',\varphi')\,dt' ,$$

where $d_\pm = 1 + i(qv_z - \Omega_\pm)\tau$. Substituting (A2) and (A4) into (A1), using $T = \Phi/\omega_c$ and the fact that φ' during the period (t_2, t_1) is less than φ' during (t_1, t) by a Bragg jump of $\pi + \Delta\varphi/4$, we have

$$\begin{aligned}\int_{-\infty}^{t} \chi(t',\varphi')\,dt' &= (\tau/d_\pm)[e^{i[(qv_z-\Omega_\pm)t \mp \varphi'] - (t-t_1)/\tau} \\ &\quad - e^{\pm i\Delta\varphi/4}(1 + e^{-d_\pm T/\tau \pm i\Delta\varphi/4})^{-1}(e^{i[(qv_z-\Omega_\pm)t_2 \mp \varphi'] - (t-t_2)/\tau} - e^{i[(qv_z-\Omega_\pm)t_1 \mp \varphi'] - (t-t_1)/\tau})] \\ &= (\tau/d_\pm)e^{i(\vec{q}'\cdot\vec{r} - \omega t \mp \varphi)}[1 - e^{-d_\pm(t_1-t)/\tau} - e^{\pm i\Delta\varphi/4}(1 + e^{-d_\pm\Phi/\omega_c\tau \pm i\Delta\varphi/4})^{-1}(e^{d_\pm(t_2-t)/\tau} - e^{d_\pm(t_1-t)/\tau})] \\ &= (\tau/d_\pm)e^{i(\vec{q}\cdot\vec{r} - \omega t \mp \varphi)}[1 - (1 + e^{\pm i\Delta\varphi/4})(1 + e^{-d_\pm\Phi/\omega_c\tau \pm i\Delta\varphi/4})^{-1} e^{d_\pm(t_1-t)/\tau}] ,\end{aligned} \tag{A5}$$

whence we obtain Eq. (5) immediately.

APPENDIX B

We here derive the spherical components of the electronic current $j_\pm = j_x \pm ij_y$ from Eq. (9) using $d^3k = m\hbar^{-2}\,d\varphi\,dk_z\,dE$ and

$$v_\pm = v_x \pm iv_y = v_\perp e^{\pm i\varphi} . \tag{B1}$$

Noting that $f_{1\pm}$ is proportional to $(1 - Q_\pm e^{-d_\pm(t-t_1)/\tau}) \times e^{\mp i\varphi}$ and that $\omega_c(t - t_1) = \varphi - \frac{1}{2}n\pi$ where $n = 0, 1, 2$, and 3 for the four sections of the hole orbit, we find

$$\int f_{1\pm} v_\mp\,d\varphi \;\alpha \sum_{n=0}^{3} \int_{n\pi/2}^{n\pi/2+\Phi} e^{\mp 2i\varphi}(1 - Q_\pm e^{-d_\pm(\varphi - n\pi/2)/\omega_c\tau})\,d\varphi$$

$$= \sum_{n=0}^{3} \int_{n\pi/2}^{n\pi/2+\Phi} e^{\mp 2i\varphi}\, d\varphi - \sum_{n=0}^{3} \int_{0}^{\Phi} e^{\mp 2i(\varphi' - n\pi/2)} Q_{\pm} e^{-d_{\pm}\varphi'/\omega_c\tau} d\varphi' = 0, \tag{B2}$$

where $\Phi = \frac{1}{2}\pi - \Delta\varphi/4$. Thus the Bragg reflections do not mix the two senses of circular polarization and we have

$$j_{\pm} = 2(2\pi)^{-3} em\hbar^{-2} \int v_{\pm} f_{1\pm}\, d\varphi\, dk_z\,, \tag{B3}$$

where we have integrated over dE and the $\partial f_0/\partial E = -\delta(E - E_F)$ factor in $f_{1\pm}$ causes the $d\varphi dk_z$ integration to be restricted to the Fermi surface. Substituting (5) in (B3) and noting from (6) and (12) that $qv_z/d_{\pm} = (1 - b_{\pm}/d_{\pm})/\tau$ we have

$$j_{\pm} = \frac{2\tau em}{(2\pi\hbar)^2} \frac{1}{2\pi} \int v_{\perp}^2 \left[\frac{e\mathcal{E}_{\pm}}{d_{\pm}} - \frac{im\omega u_{\perp}}{2\tau} \left(1 - \frac{b_{\pm}}{d_{\pm}}\right) \right] \times \left[1 - Q_{\pm} e^{-d_{\pm}(t-t_1)/\tau}\right] d\varphi\, dk_z\,, \tag{B4}$$

where we have dropped a factor of $e^{i(\vec{q}\cdot\vec{R} - \omega t)}$. The entire φ dependence of the integrand is in $(t - t_1)$, which may be replaced by φ/ω_c and integrated over $0 \le \varphi \le \Phi$ in each of the four sections of the hole orbit (for electron orbits there is no φ dependence and a factor 2π is obtained from the φ integration) to yield Eq. (10).

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[9]See the Appendix in I.

[10]Consider, for instance, an intrinsic semiconductor. The current associated with a long-wavelength phonon is negligible because the valence electrons follow the ions in their motion even though those same electrons give no contribution to the zero-frequency conductivity.

[11]Besides the usually negligible contribution mentioned above collision drag also enters the jellium calculation through the distribution function.

[12]We have corrected an unimportant sign mistake in (15).

[13]From Figs. 1(a) and 1(b) one easily sees that $\Delta\varphi/4 = 2\cos^{-1}(K/2k_{\perp})$, where $k_{\perp}^2 = k_F^2 - k_z^2$.

[14]The -1 in $N(1/G - 1)$ is important only for large fields where it makes the attenuation approach zero.

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Density of Electronic States in Liquid Aluminum

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The density-matrix approach due to March and his collaborators is used to obtain the electronic density of states of liquid metals. A model potential is used for liquid aluminum, and the density of states is calculated for both completely random and correlated systems. Results are compared with previous calculations, and nearly-free-electron-like behavior is found for liquid aluminum.

I. INTRODUCTION

In recent years the study of electronic states in liquid metals[1] has received considerable attention. The effort has been mostly directed towards developing formal techniques for tackling the problem of cellular disorder. There have been rather few attempts to evaluate the theoretical expressions numerically for real systems.

Ballentine[2] has calculated the density of states in several liquid metals by using the Green's-function method of Edwards.[3] He used a local energy-independent Heine-Abarenkov-type potential. Ballentine used Edwards's theory but replaced the